(1) (30%) Let \( a \) be a positive real number, and let the sequence of real numbers \( x_i \) be given by
\[
x_0 = 1, \quad x_{i+1} = \frac{1}{2} \left( x_i + \frac{a}{x_i} \right) \quad \text{for } i = 0, 1, 2, \ldots
\]
It can be shown mathematically that \( x_i \to \sqrt{a} \) as \( i \to \infty \)
This algorithm is derived from the Newton-Raphson method in numerical analysis. Write a program that reads in the value of \( a \) interactively and uses this algorithm to compute the sequence root of \( a \). As you will see, the program is very efficient. (Nonetheless, it is not the algorithm used by the sqrt() function in the standard library.)
Declare \( x_0 \) and \( x_1 \) to be of type double, and initialize \( x_1 \) to be 1. Inside a loop do the following
\[
x_0 = x_1; \quad /* \text{save the current value of } x_1 */
x_1 = 0.5 * (x_1 + a / x_1); \quad /* \text{compute a new value of } x_1 */
\]
Each time through the loop, print out the values of \( x_i \)

(2) (30%) The constant \( e \), which is the base of the natural logarithms, is given to 41 significant figures by
\[
e = 2.71828 \ 18284 \ 59045 \ 23536 \ 02874 \ 71352 \ 66249 \ 77572
\]
Define
\[
x_n = (1 + \frac{1}{n})^n \quad \text{for } n = 1, 2, \ldots
\]
It can be shown mathematically that \( x_n \to e \) as \( n \to \infty \)
Investigate how to calculate \( e \) to arbitrary precision using this algorithm. You will find that the algorithm is computationally ineffective. (See exercise 36, on page 195)

(3) (40%) In addition to the algorithm given in the previous exercise, the value for \( e \) is also given by the infinite series
\[
e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots
\]
The above algorithm is computationally effective. Use it to compute \( e \) to an arbitrary precision.

(4) Update your report to server, otherwise you will get -10 point.
(5) If you will not submit your report, you get 0 point.
**Command Line:** (You must use Parameter argc and argv)

Q1: ./hw2_1 a i (Please Follow this Sequence, otherwise you will get -20 point)
Q2: ./hw2_2 n
Q3: ./hw2_3 n

**Output:**

Q1: print result $X_i$ for top-i loop (get the ten digit after the point)
Q2: print result $X_n$ for top-n loop (get the ten digit after the point)
Q3: print result $X_n$ for top-n loop (get the ten digit after the point)

**Example**

```bash
> ./hw2_1 2 2
> 1.5000000000
> 1.4166666666
> ./hw2_2 2
> 2.0000000000
> 2.2500000000
> ./hw2_3 2
> 2.0000000000
> 2.5000000000
```

**Report**

除了要交紙本，也要將 report 以電子檔的形式上傳至 server

電子檔案名格式：HW2.docx

以下為教學，若仍有許多不懂的地方，下次上課後助教再實際操作一次

1. 安裝 FileZilla
2. 連線方式：打開 FileZilla 後>檔案>站台管理員
3. 如圖設定好連線方式後連線
4. 這邊直接確認就好

5. 進到對應的目錄並上傳你的電子檔，一份作業一個電子檔